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Josephson Current between Triplet and Singlet Superconductors

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(Received February 1, 2008)

The Josephson effect between triplet and singlet superconductors is studied. Josephson current can flow between triplet and singlet superconductors due to the spin-orbit coupling in the spin-triplet superconductor but it is finite only when triplet superconductor has $L_z = -S_z = \pm 1$, where L_z and S_z are the perpendicular components of orbital angular momentum and spin angular momentum of the triplet Cooper pairs, respectively. The recently observed temperature and orientational dependence of the critical current through a Josephson junction between UPt_3 and Nb is investigated by considering a non-unitary triplet state.

KEYWORDS: triplet superconductor, odd-parity, spin-orbit coupling Josephson junction

Superconductivity in UPt_3 attracts a lot of interests. The existence of two different superconducting phases (A and B phases) as well as the third phase in the magnetic field (C phase) shows that the superconductivity in this system is not a conventional s-wave spin-singlet superconductivity. The possibility of the triplet superconductivity has been discussed.[1,2,3,4] The anisotropy of H_{c2}[5] is discussed as an evidence for the triplet superconductivity with the $\boldsymbol{d}$ vector parallel to the c-axis.[2] Experimental results of the Knight shift observed by the NMR[6,7,8] and μSR[9] show the evidence for the triplet superconductivity in UPt_3.[10] The Knight shift is observed to be independent of temperature in the superconducting phase for $H > 5$ kOe.[6,7,8] It decreases below T_c^+ (A and B phases) for $H < 5$ kOe when $H \parallel \hat{\boldsymbol{b}}$ and below T_c^- (B phase) for $H < 2.3$ kOe when $H \parallel \hat{\boldsymbol{c}}$.[8] The spin susceptibility for the spin-singlet superconductivity decreases as temperature becomes low, while that for the spin-triplet superconductivity is independent of temperature if the magnetic field is perpendicular to the $\boldsymbol{d}$ vector. Therefore, the magnetic-field and temperature dependences of the Knight sift of UPt_3 can be understood by the spin triplet states. The A and B phases are identified with $\boldsymbol{d}(\boldsymbol{k}) = d_b(\boldsymbol{k})\hat{\boldsymbol{b}}$ and $\boldsymbol{d}(\boldsymbol{k}) = d_b(\boldsymbol{k})\hat{\boldsymbol{b}} + \mathrm{i}d_c(\boldsymbol{k})\hat{\boldsymbol{c}}$, respectively.[8,11,10]

Since the Josephson effect is controlled by both the amplitude and the phase of the order parameters, it is a powerful tool to identify the symmetry of the order parameters. Recently, Sumiyama *et al.*[12] have observed the Josephson critical current between a single crystal UPt_3 and an s-wave superconductor Nb. They also observed the Shapiro steps of the $\hbar\omega/(2e)$. If the Josephson current is caused by the higher-order tunneling or the proximity induced tunneling, the steps of $\hbar\omega/(2ne)$ with $n \geq 2$ should be observed.[13,14] Thus it is concluded that the Josephson current between UPt_3 and Nb is caused by the ordinary pair tunneling of the Cooper pairs. They found that the critical current is very small in the A phase and it increases steeply as temperature becomes lower than T_c^- for the current parallel to the c-axis. The Josephson current along the b-axis is observed even above T_c^- and the increasing rate of the critical current as decreasing temperature becomes slower below T_c^-.

The order parameters near the interface may be different from those in the bulk. Recently, it has been shown that the localized zero energy state exists at the interface when the order parameter change sign through the reflection.[15] The zero energy state has been studied for the d-wave superconductors[15,16,17,18] and also for the triplet superconductor.[19] Although the zero energy state should have an influence on the Josephson current, most of the essential features of the Josephson current can be understood by considering the lowest order in the tunneling Hamiltonian for the uniform order parameters in both singlet and triplet superconductors.

The Josephson current between spin triplet and spin singlet superconductors is forbidden if tunneling Hamiltonian does not change the spin.[20] Due to the spin-orbit coupling in UPt_3, the spin-triplet parings are actually the odd-parity parings, i.e. a triplet state is formed in the pseudospin, which is a superposition of the states with different spins. Although the temperature independent Knight shift for $H > 5$ kOe suggests a weak spin-orbit coupling in UPt_3, the spin-orbit coupling cannot be neglected. Fenton[21] and Geshkenbein and Larkin[22] have shown that the Josephson effect between triplet and singlet superconductors can occur due to the spin-orbit coupling. The Josephson tunneling between conventional and unconventional superconductors are also studied by several authors,[23,24,13,25] but they assumed the unitary states for the triplet superconductors.

In this letter we consider a Josephson current between triplet and singlet superconductors with the spin-orbit coupling in the triplet superconductor by taking account of the non-unitary triplet superconductivity. We write the wave number, spin and the creation operator for the triplet (singlet) superconductor as $\boldsymbol{k}$, μ and $a^\dagger_{\boldsymbol{k},\mu}$ ($\boldsymbol{l}$, ν and $b^\dagger_{\boldsymbol{l},\nu}$), respectively. Then the tunneling Hamiltonian

in the presence of spin-orbit coupling is given by

$$\mathcal{H}_T = \sum_{\boldsymbol{k},\boldsymbol{l}} \sum_{\mu,\nu} \{T_{\mu,\nu}(\boldsymbol{k},\boldsymbol{l}) a^\dagger_{\boldsymbol{k},\mu} b_{\boldsymbol{l},\nu} + h.c.\}, \tag{1}$$

where

$$T_{\mu\nu}(\boldsymbol{k},\boldsymbol{l}) = T(\boldsymbol{k},\boldsymbol{l})\delta_{\mu\nu} + T'(\boldsymbol{k},\boldsymbol{l})(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{n}})\cdot\vec{\sigma}_{\mu\nu}. \tag{2}$$

The first term in eq.(2) is the spin diagonal matrix element, and the second term is the tunneling matrix element due to the spin-orbit coupling,[22] where $\hat{\boldsymbol{n}}$ is a unit vector normal to the interface, $\hat{\boldsymbol{k}} = \boldsymbol{k}/k_F$ (Fermi surface is assumed to be spherical), and $\vec{\sigma} = (\sigma^x, \sigma^y, \sigma^z)$ is a vector with Pauli matrices. In the second order perturbation in $\mathcal{H}_T$ the Josephson coupling energy is calculated as

$$\Delta E = \langle 0|\mathcal{H}_T \frac{1}{E_0 - \mathcal{H}_0}\mathcal{H}_T|0\rangle, \tag{3}$$

where $\mathcal{H}_0$ is a direct sum of the Hamiltonian for two superconductors and E_0 is the ground state energy for $\mathcal{H}_0$. The Josephson current is calculated as[26]

$$\begin{aligned} I = &-e\langle 0|\dot{N}_a \frac{1}{E_0 - \mathcal{H}_0}\mathcal{H}_T|0\rangle \\ &-e\langle 0|\mathcal{H}_T \frac{1}{E_0 - \mathcal{H}_0}\dot{N}_a|0\rangle, \end{aligned} \tag{4}$$

where

$$N_a = \sum_{\boldsymbol{k}} \sum_{\mu} a^\dagger_{\boldsymbol{k},\mu} a_{\boldsymbol{k},\mu}, \tag{5}$$

and

$$\begin{aligned} \dot{N}_a &= \frac{\mathrm{i}}{\hbar}[N_a, \mathcal{H}_T] \\ &= -\frac{\mathrm{i}}{\hbar}\sum_{\boldsymbol{k},\boldsymbol{l}}\sum_{\mu,\nu}\{T_{\mu,\nu}(\boldsymbol{k},\boldsymbol{l}) a^\dagger_{\boldsymbol{k},\mu} b_{\boldsymbol{l},\nu} - h.c.\}. \end{aligned} \tag{6}$$

The Bogoliubov transformations for triplet and singlet superconductors are written as

$$a_{\boldsymbol{k}\mu} = \sum_{\mu'} \left(u^{(t)}_{\boldsymbol{k}\mu\mu'} \alpha_{\boldsymbol{k}\mu'} + v^{(t)}_{\boldsymbol{k}\mu\mu'} \alpha^\dagger_{-\boldsymbol{k}\mu'} \right), \tag{7}$$

and

$$b_{\boldsymbol{l}\nu} = \sum_{\nu'} \left(u^{(s)}_{\boldsymbol{l}\nu\nu'} \beta_{\boldsymbol{l}\nu'} + v^{(s)}_{\boldsymbol{l}\nu\nu'} \beta^\dagger_{-\boldsymbol{l}\nu'} \right). \tag{8}$$

For the singlet superconductor the order parameter is written as

$$\Delta^{(s)}_{\nu\nu'}(\boldsymbol{l}) = \Psi(\boldsymbol{l})\,(\mathrm{i}\sigma^y)_{\nu\nu'}, \tag{9}$$

and the Bogoliubov transformation is given by

$$\begin{aligned} u^{(s)}_{\boldsymbol{l}\nu\nu'} &= \sqrt{\frac{E_{\boldsymbol{l}} + \epsilon_{\boldsymbol{l}}}{2E_{\boldsymbol{l}}}}\delta_{\nu\nu'} \\ v^{(s)}_{\boldsymbol{l}\nu\nu'} &= \frac{(-\mathrm{i}\sigma_y)_{\nu\nu'}\Psi(\boldsymbol{l})}{\sqrt{2E_{\boldsymbol{l}}(E_{\boldsymbol{l}} + \epsilon_{\boldsymbol{l}})}}, \end{aligned} \tag{10}$$

where

$$E_{\boldsymbol{l}} = \sqrt{\epsilon^2_{\boldsymbol{l}} + |\Psi(\boldsymbol{l})|^2}. \tag{11}$$

The order parameter for spin triplet is written as

$$\Delta^{(t)}_{\mu\mu'}(\mathbf{k}) = (\mathrm{i}(\vec{\sigma}\cdot\boldsymbol{d}(\boldsymbol{k}))\sigma^y)_{\mu\mu'}. \tag{12}$$

If $\boldsymbol{d}(\boldsymbol{k})\times\boldsymbol{d}^*(\boldsymbol{k}) = 0$, the order parameter given in eq. (12) is proportional to a unitary matrix and it is called as a unitary state. If $\boldsymbol{d}(\boldsymbol{k})\times\boldsymbol{d}^*(\boldsymbol{k}) \neq 0$, the state is called non-unitary. For the unitary state the Bogoliubov transformation is given by[3]

$$\begin{aligned} u^{(t)}_{\boldsymbol{k}\mu\mu'} &= \sqrt{\frac{E_{\boldsymbol{k}} + \epsilon_{\boldsymbol{k}}}{2E_{\boldsymbol{k}}}}\delta_{\mu,\mu'} \\ v^{(t)}_{\boldsymbol{k}\mu\mu'} &= \frac{(-\mathrm{i}(\boldsymbol{d}(\boldsymbol{k})\cdot\vec{\sigma})\sigma_y)_{\mu\mu'}}{\sqrt{2E_{\boldsymbol{k}}(E_{\boldsymbol{k}} + \epsilon_{\boldsymbol{k}})}} \end{aligned} \tag{13}$$

where

$$E_{\boldsymbol{k}} = \sqrt{\epsilon^2_{\boldsymbol{k}} + |\boldsymbol{d}(\boldsymbol{k})|^2}. \tag{14}$$

We get

$$\Delta E = 2\mathrm{Re}\sum_{\boldsymbol{kl}} \frac{T(\boldsymbol{k},\boldsymbol{l})T'(-\boldsymbol{k},-\boldsymbol{l})(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{n}})\cdot\boldsymbol{d}^*(\boldsymbol{k})\Psi(\boldsymbol{l})}{E_{\boldsymbol{k}}E_{\boldsymbol{l}}(E_{\boldsymbol{k}} + E_{\boldsymbol{l}})} \tag{15}$$

In the above we have took only terms depending on the phase difference of the order parameters and we have neglected the phase independent terms which are irrelevant to the Josephson critical current.

The Josephson current between the spin-triplet unitary state and spin singlet state is obtained as[22]

$$I = \frac{-2e}{\hbar}\mathrm{Im}\sum_{\boldsymbol{kl}} \frac{T(\boldsymbol{k},\boldsymbol{l})T'(-\boldsymbol{k},-\boldsymbol{l})(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{n}})\cdot\boldsymbol{d}^*(\boldsymbol{k})\Psi(\boldsymbol{l})}{E_{\boldsymbol{k}}E_{\boldsymbol{l}}(E_{\boldsymbol{k}} + E_{\boldsymbol{l}})} \tag{16}$$

From eq.(16) it is obtained that the Josephson current is zero if $\boldsymbol{d}(\boldsymbol{k})$ is parallel to $\boldsymbol{n}$. Therefore, if $\boldsymbol{d}$ vector is parallel to the c-axis, the Josephson current along the c-axis cannot be explained. Even if $\boldsymbol{d}(\boldsymbol{k})$ is not parallel to $\hat{\boldsymbol{c}}$, the Josephson current is zero for the states belonging to the one-dimensional representation of the D_{6h} symmetry for the weak spin-orbit coupling case such as $\boldsymbol{d}(\boldsymbol{k}) \propto k_c\hat{\boldsymbol{b}}$ or $\boldsymbol{d}(\boldsymbol{k}) \propto k_c(k_b^2 + k_b^2)\hat{\boldsymbol{b}}$ due to the summation over $\boldsymbol{k}$, since $T(\boldsymbol{k},\boldsymbol{l})$ and $T'(\boldsymbol{k},\boldsymbol{l})$ have six-fold rotational symmetry for $\boldsymbol{k}$ in the a-b plane in UPt_3. These states have finite Josephson current along the a-axis. This can be understood as follows. Since the total (orbital (L) plus spin (S)) angular momentum is conserved by the tunneling Hamiltonian, only the Cooper pairs with angular momentum $L_z = -S_z = \pm 1$ can be transformed into spin-singlet pairs. If $\boldsymbol{d}(\boldsymbol{k}) \parallel \hat{\boldsymbol{c}}$, Cooper pairs are formed in the states of $S_z = 0$, and no Josephson current flows. Finite Josephson current parallel to the c-axis is possible for the state $\boldsymbol{d}(\boldsymbol{k})\times\hat{\boldsymbol{c}} \neq 0$ such as

1. $\boldsymbol{d}(\boldsymbol{k}) \propto k_c^2(k_a + \mathrm{i}k_b)\hat{\boldsymbol{b}}$,
2. $\boldsymbol{d}(\boldsymbol{k}) \propto (k_c^2 - C)(k_a + \mathrm{i}k_b)\hat{\boldsymbol{b}}$,
3. $\boldsymbol{d}(\boldsymbol{k}) \propto (k_c^2 - C)(k_a)\hat{\boldsymbol{b}}$,

etc., where C is a constant. The c component of the angular momentum is 1 for the first and the second examples and superposition of 1 and -1 for the third example. These states belong to the two-dimensional representation for the weak spin-orbit coupling case, since $L_z = 1$

and -1 make the basis for the two-dimensional representation. The z component of the spin of the Cooper pair is a superposition of 1 and -1 in these cases of the unitary states. As a result Josephson current is not forbidden by the symmetry. The energy gap is zero at the equator ($k_c = 0$) and the poles ($k_a = k_b = 0$) of the Fermi surface for the first example, at the parallels of latitude ($k_c = \pm\sqrt{C}$) and poles for the second example, and at the parallels of latitude and lines of longitude ($k_a = 0$) for the third example. The density of states $N(E)$ for low energy excitations is proportional to $\sqrt{E}$, E, and $E\log(1/E)$ for three examples, respectively. The Josephson critical current perpendicular to the c-axis is zero for the above three cases.

Next we consider non-unitary states. For simplicity we take $-\mathrm{i}\boldsymbol{d}(\boldsymbol{k}) \times \boldsymbol{d}^*(\boldsymbol{k}) \parallel \hat{\boldsymbol{z}}$, which is equivalent to $d_z(\boldsymbol{k}) = 0$. This choice of the quantization direction for the spin is possible for the equal-spin-paring state, i.e., only up spins ($S_z = 1$) and down spins ($S_z = -1$) make Cooper pairs and no up-down pairs ($S_z = 0$) exist by suitably chosen quantization axis in the spin space. Note the z direction in the spin space is not necessarily to be parallel to the c-axis in the real space for the weak spin-orbit coupling case. Then the Bogoliubov transformation is given by

$$u^{(t)}_{\boldsymbol{k}\mu\mu'} = \sqrt{\frac{E_{\boldsymbol{k}\mu'} + \epsilon_{\boldsymbol{k}}}{2E_{\boldsymbol{k}\mu'}}}\delta_{\mu\mu'} \tag{17}$$

$$v^{(t)}_{\boldsymbol{k}\mu\mu'} = \frac{(-\mathrm{i}(\boldsymbol{d}(\boldsymbol{k})\cdot\vec{\sigma})\sigma_y)_{\mu\mu'}}{\sqrt{2E_{\boldsymbol{k}\mu'}(E_{\boldsymbol{k}\mu'} + \epsilon_{\boldsymbol{k}})}} \tag{18}$$

where

$$E_{\boldsymbol{k}\pm} = \sqrt{\epsilon^2_{\boldsymbol{k}} + |\boldsymbol{d}(\boldsymbol{k})|^2 \pm |\boldsymbol{d}(\boldsymbol{k}) \times \boldsymbol{d}^*(\boldsymbol{k})|}. \tag{19}$$

We get the Josephson coupling energy for the non-unitary spin-triplet and spin-singlet superconductors as

$$\begin{aligned}\Delta E = 2\mathrm{Re}\sum_{\boldsymbol{k},\boldsymbol{l}} T(\boldsymbol{k},\boldsymbol{l})T'(-\boldsymbol{k},-\boldsymbol{l}) \\ \times \left[\frac{\left((\hat{\boldsymbol{k}}\times\hat{\boldsymbol{n}})\cdot\boldsymbol{d}^*(\boldsymbol{k}) + \left(\mathrm{i}(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{n}})\times\boldsymbol{d}^*(\boldsymbol{k})\right)_z\right)\Psi(\boldsymbol{l})}{2(E_{\boldsymbol{k}+}+E_{\boldsymbol{l}})E_{\boldsymbol{k}+}E_{\boldsymbol{l}}}\right. \\ + \left.\frac{\left((\hat{\boldsymbol{k}}\times\hat{\boldsymbol{n}})\cdot\boldsymbol{d}^*(\boldsymbol{k}) - \left(\mathrm{i}(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{n}})\times\boldsymbol{d}^*(\boldsymbol{k})\right)_z\right)\Psi(\boldsymbol{l})}{2(E_{\boldsymbol{k}-}+E_{\boldsymbol{l}})E_{\boldsymbol{k}-}E_{\boldsymbol{l}}}\right].\end{aligned} \tag{20}$$

The Josephson current between non-unitary spin-triplet and spin singlet superconductors is obtained as

$$\begin{aligned}I = \frac{-2e}{\hbar}\mathrm{Im}\sum_{\boldsymbol{k},\boldsymbol{l}} T(\boldsymbol{k},\boldsymbol{l})T'(-\boldsymbol{k},-\boldsymbol{l}) \\ \times \left[\frac{\left((\hat{\boldsymbol{k}}\times\hat{\boldsymbol{n}})\cdot\boldsymbol{d}^*(\boldsymbol{k}) + \left(\mathrm{i}(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{n}})\times\boldsymbol{d}^*(\boldsymbol{k})\right)_z\right)\Psi(\boldsymbol{l})}{2(E_{\boldsymbol{k}+}+E_{\boldsymbol{l}})E_{\boldsymbol{k}+}E_{\boldsymbol{l}}}\right. \\ + \left.\frac{\left((\hat{\boldsymbol{k}}\times\hat{\boldsymbol{n}})\cdot\boldsymbol{d}^*(\boldsymbol{k}) - \left(\mathrm{i}(\boldsymbol{k}\times\hat{\boldsymbol{n}})\times\boldsymbol{d}^*(\hat{\boldsymbol{k}})\right)_z\right)\Psi(\boldsymbol{l})}{2(E_{\boldsymbol{k}-}+E_{\boldsymbol{l}})E_{\boldsymbol{k}-}E_{\boldsymbol{l}}}\right].\end{aligned} \tag{21}$$

By noting that

$$\begin{aligned}(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{n}})\cdot\boldsymbol{d}^*(\boldsymbol{k}) \pm \mathrm{i}\left((\hat{\boldsymbol{k}}\times\hat{\boldsymbol{n}})\times\boldsymbol{d}^*(\boldsymbol{k})\right)_z \\ = [(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{n}})_x \mp \mathrm{i}(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{n}})_y][d^*_x(\boldsymbol{k}) \pm \mathrm{i}d^*_y(\boldsymbol{k})],\end{aligned} \tag{22}$$

we obtain that the Josephson current along the z-axis is zero for the states with $L_z = S_z = 1$, which are the case for example in $\boldsymbol{d}(\boldsymbol{k}) \propto (k_x + \mathrm{i}k_y)(\hat{\boldsymbol{x}} + \mathrm{i}\hat{\boldsymbol{y}})$ or $\boldsymbol{d}(\boldsymbol{k}) \propto (k_z^2 - C)(k_x + \mathrm{i}k_y)(\hat{\boldsymbol{x}} + \mathrm{i}\hat{\boldsymbol{y}})$. In the non-unitary states with $L_z = -S_z = 1$ such as $(k_x + \mathrm{i}k_y)(\hat{\boldsymbol{x}} - \mathrm{i}\hat{\boldsymbol{y}})$ or $(k_z^2 - C)(k_x + \mathrm{i}k_y)(\hat{\boldsymbol{x}} - \mathrm{i}\hat{\boldsymbol{y}})$ only the electrons with down spin form Cooper pairs and the Josephson current can be finite along the z-axis.

Here we assume the B phase of $\mathrm{UPt_3}$ as a non-unitary state with

$$\boldsymbol{d}(\boldsymbol{k}) = d_b(\boldsymbol{k})\hat{\boldsymbol{b}} + id_c(\boldsymbol{k})\hat{\boldsymbol{c}}, \tag{23}$$

as proposed to explain the Knight shift.[8,10] We take $\hat{\boldsymbol{a}}$, $\hat{\boldsymbol{b}}$ and $\hat{\boldsymbol{c}}$ as $\hat{\boldsymbol{z}}$, $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ respectively in order to have $-\mathrm{i}\boldsymbol{d}(\boldsymbol{k}) \times \boldsymbol{d}^*(\boldsymbol{k}) \parallel \hat{\boldsymbol{z}}$. We take $d_c(\boldsymbol{k})/d_a(\boldsymbol{k})$ to be real and we get

$$E_{\boldsymbol{k}\pm} = \sqrt{\epsilon^2_{\boldsymbol{k}} + |d_b(\boldsymbol{k}) \pm d_c(\boldsymbol{k})|^2}. \tag{24}$$

For the c-axis junction we put $\hat{\boldsymbol{n}} = \hat{\boldsymbol{y}} = \hat{\boldsymbol{c}}$ and get the Josephson current as

$$\begin{aligned}I_c = \frac{-2e}{\hbar}\mathrm{Im}\sum_{\boldsymbol{k},\boldsymbol{l}} T(\boldsymbol{k},\boldsymbol{l})T'(-\boldsymbol{k},-\boldsymbol{l}) \\ \times \left[\frac{-k_a(d^*_b(\boldsymbol{k}) + d^*_c(\boldsymbol{k}))\Psi(\boldsymbol{l})}{2(E_{\boldsymbol{k}+}+E_{\boldsymbol{l}})E_{\boldsymbol{k}+}E_{\boldsymbol{l}}}\right. \\ + \left.\frac{-k_a(d^*_b(\boldsymbol{k}) - d^*_c(\boldsymbol{k}))\Psi(\boldsymbol{l})}{2(E_{\boldsymbol{k}-}+E_{\boldsymbol{l}})E_{\boldsymbol{k}-}E_{\boldsymbol{l}}}\right].\end{aligned} \tag{25}$$

For the b-axis junction ($\hat{\boldsymbol{n}} = \hat{\boldsymbol{x}} = \hat{\boldsymbol{b}}$), we get the Josephson current as

$$\begin{aligned}I_b = \frac{-2e}{\hbar}\mathrm{Im}\sum_{\boldsymbol{k},\boldsymbol{l}} T(\boldsymbol{k},\boldsymbol{l})T'(-\boldsymbol{k},-\boldsymbol{l}) \\ \times \left[\frac{-\mathrm{i}k_a(d^*_b(\boldsymbol{k}) + d^*_c(\boldsymbol{k}))\Psi(\boldsymbol{l})}{2(E_{\boldsymbol{k}+}+E_{\boldsymbol{l}})E_{\boldsymbol{k}+}E_{\boldsymbol{l}}}\right. \\ + \left.\frac{\mathrm{i}k_a(d^*_b(\boldsymbol{k}) - d^*_c(\boldsymbol{k}))\Psi(\boldsymbol{l})}{2(E_{\boldsymbol{k}-}+E_{\boldsymbol{l}})E_{\boldsymbol{k}-}E_{\boldsymbol{l}}}\right].\end{aligned} \tag{26}$$

The Josephson current for the a-axis junction ($\hat{\boldsymbol{n}} = \hat{\boldsymbol{z}} = \hat{\boldsymbol{a}}$) is given by

$$I_a = \frac{-2e}{\hbar}\mathrm{Im}\sum_{\boldsymbol{k},\boldsymbol{l}} T(\boldsymbol{k},\boldsymbol{l})T'(-\boldsymbol{k},-\boldsymbol{l})$$

$$\times \left[\frac{\mathrm{i}(k_b - \mathrm{i}k_c)(d_b^*(\boldsymbol{k}) + d_c^*(\boldsymbol{k}))\Psi(\boldsymbol{l})}{2(E_{\boldsymbol{k}+} + E_{\boldsymbol{l}})E_{\boldsymbol{k}+}E_{\boldsymbol{l}}} + \frac{-\mathrm{i}(k_b + \mathrm{i}k_c)(d_b^*(\boldsymbol{k}) - d_c^*(\boldsymbol{k}))\Psi(\boldsymbol{l})}{2(E_{\boldsymbol{k}-} + E_{\boldsymbol{l}})E_{\boldsymbol{k}-}E_{\boldsymbol{l}}} \right]. \tag{27}$$

If we take $d_b(\boldsymbol{k}) \neq 0$ for $T < T_c^+$ (A and B phases) and $d_c(\boldsymbol{k}) \neq 0$ for $T < T_c^-$ (B phase), it will be difficult to explain the experimental results that the Josephson critical current is very small in the A phase and increases steeply below T_c^- for $I \parallel c$ while I_b increases more slowly below T_c^- than in the A phase.[12)]

On the other hand if we assume $d_c(\boldsymbol{k}) \propto k_a$, $(k_c^2 - C)k_a$, $k_a + \mathrm{i}k_b$, or $(k_c^2 - C)(k_a + \mathrm{i}k_b)$ becomes finite below T_c^+ and $d_b(\boldsymbol{k})$ becomes finite below T_c^- in the same $\boldsymbol{k}$ dependence as $d_c(\boldsymbol{k})$, we get I_c is zero for $T_c^- < T < T_c^+$ and it increases as $d_b^*(\boldsymbol{k})\Psi(\boldsymbol{l})$ for $T < T_c^-$. In this assumption we get I_b is finite below T_c^+. Below T_c^- I_b is not changed in the linear order in $|d_b(\boldsymbol{k})|$ but it changes in the higher order in $|d_b(\boldsymbol{k})|$ and the second term in eq.(26) becomes small when $d_b(\boldsymbol{k}) \approx d_c(\boldsymbol{k})$. Thus we expect the slower increase of I_b below T_c^-.

In conclusion we have shown that the Josephson current between triplet and singlet superconductors is allowed if the Cooper pairs in the triplet superconductors have a component $L_z = -S_z = \pm 1$, where z is the direction normal to the junction. These results are obtained by the lowest order perturbation in the tunneling Hamiltonian at $T = 0$, but the extension to a finite temperature is possible as in the Junctions between singlet superconductors.[26)] We can understand the temperature and orientational dependence of the Josephson critical current[12)] by assuming the non-unitary state in the B phase, which is proposed to explain the Knight shift in NMR,[8, 10)] although we have used the different $\boldsymbol{d}$ vector in the A phase from that is used to explain the Knight shift.

The author would like to thank A. Sumiyama for showing the experimental data prior to publication and useful discussions.

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